NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 07BAMS	**LEVEL:** 6
COURSE CODE: LIA601S	**COURSE NAME:** LINEAR ALGEBRA
SESSION: JULY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

SUPPLEMENTARY/ SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	DR. NA CHERE
MODERATOR:	DR. DSI IIYAMBO

INSTRUCTIONS
1. Answer ALL the questions in the booklet provided. 2. Show clearly all the steps used in the calculations. 3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

QUESTION 1 [12]

For each of the following questions, state whether it is true or false. Justify or give a counter example if your answer is false.

1.1. The mapping $T: \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(v) = v + v_0$ for all v in $\mathbb{R}^n$ and v_0 a non-zero fixed vector in $\mathbb{R}^n$ is linear. [3]

1.2. A square matrix A is invertible if and only if 0 is an eigenvalue of A. [3]

1.3. If A is an n x n matrix, then the geometric multiplicity of each eigenvalue is less than or equal to its algebraic multiplicity. [2]

1.4. If two matrices of the same size have the same determinant then they are similar. [4]

QUESTION 2 [35]

Let $T: P_2 \to \mathbb{R}^2$ be a mapping defined by $T(a_0 + a_1x + a_2x^2) = \begin{bmatrix} a_0 - a_1 \\ a_1 + a_2 \end{bmatrix}$.

2.1. Show that T is linear. [12]

2.2. Determine the bases for the kernel and range of T. [14]

2.3. Is T singular or nonsingular? Explain. [3]

2.4. State the nullity and rank of T and verify the dimension theorem. [6]

QUESTION 3 [8]

Let $\mathcal{B} = \{v_1, v_2\}$ and $C = \{u_1, u_2\}$ be bases for a vector space V and suppose $v_1 = 2u_1 + 3u_2$ and $v_2 = 5u_1 - 6u_2$.

3.1. Find the change of basis matrix from $\mathcal{B}$ to C $(P_{C\leftarrow B})$. [4]

3.2. Use the result in part (3.1) to find $[x]_C$ for $x = 3v_1 - 2v_2$. [4]

QUESTION 4 [11]

Consider the bases $B = \{1 + x + x^2, x + x^2, x^2\}$ and $C = \{1, x, x^2\}$ of P_2.

4.1. Find the change of basis matrix $P_{B\leftarrow C}$ from C to B. [8]

4.2. Use the result in part (4.1) to compute $[p(x)]_B$ where $p(x) = 2 + x - 3x^2$. [3]

QUESTION 5 [27]

Let T be a linear operator on $\mathbb{R}^3$ defined by $\mathrm{T}(\mathrm{x}, \mathrm{y}, \mathrm{z}) = (2\mathrm{x} - \mathrm{y} - \mathrm{z},\ \mathrm{x} - \mathrm{z}, -\mathrm{x} + \mathrm{y} + 2\mathrm{z})$.

5.1. Find the matrix of T with respect to the standard basis vectors of $\mathbb{R}^3$. [6]

5.2. Find the eigenvalues and the corresponding eigenspaces of the linear operator T. [21]

QUESTION 6 [7]

Find the quadratic form $\mathrm{q}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3)$ for the symmetric matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$.

END OF SECOND/SUPPLEMENTARY EXAMINATION QUESTION PAPER